IMPEDEANCE MATCHING 101

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WHY IMPEDANCE MATCH?

FOR TRANSMITTING - Impedances are necessarily matched to enable efficient power transfer from one device to another, such as an output amplifier to an antenna or a driver amplifier to an output amplifier.

FOR RECEIVING – A low noise preamplifier is designed to achieve its minimum noise figure for a particular source or driving impedance. The antenna must be matched to this impedance to achieve the minimum noise performance.
THE CASE FOR 50 OHMS

- A common impedance used for Amateur Radio applications is 50 ohms.
- The feedlines, i.e. the coax, are designed for 50 ohms, a common standard impedance.
- The power amplifiers are designed to operate into a 50 ohm load, i.e. the antenna fed with 50 ohm coax.
- The receiver preamps are designed to achieve minimum noise figure with a 50 ohm driving source; i.e. the antenna/feedline.
THE THREE ELEMENTS OF IMPEDANCES

1. The resistor, $R$

2. The inductor, $L$

3. The capacitor, $C$
THE BASICS,
THE IMPEDANCE OF EACH COMPONENT

\[ Z = R \]

\[ Z = +jX_L \quad |Z| = |X_L| \]

\[ Z = -jX_C \quad |Z| = |X_C| \]
THE IMPEDANCE OF EACH COMPONENT

\[ |X_L| = 2\pi f L, \]
where: \( f \) = frequency in Hz
\( L \) = inductance in henries

\[ |X_C| = 1/(2\pi f C) \]
where: \( f \) = frequency in Hz
\( C \) = capacitance in farads

and \( Z = R \pm jX \)
THE COMPLEX IMPEDANCE

\[ Z = R + jX \]

\[ Z = R - jX \]
EXPRESS \( f \), \( L \), AND \( C \) USING SCIENTIFIC NOTATION

**SHORT CUTS**

\[
\begin{align*}
x \ 1000 &= x \ 10^3 \\
1/1000 &= x \ 10^{-3}
\end{align*}
\]

\[
\begin{align*}
2\pi &= 6.28 \\
1/(2\pi) &= 0.159 = 159 \times 10^{-3}
\end{align*}
\]

\( f \) in MHz = \( x \ 10^6 \)

(i.e 144 MHz = 144 \times 10^6 Hz)

10\(^{-9}\) henries = nanohenries, nH

10\(^{-12}\) farads = picofarads, pf
The loaded $Q = \frac{X_s}{R_s} = \frac{R_p}{X_p}$

A series R & X circuit has an equivalent parallel R & X circuit.

$X_s$ is the series reactance, $R_s$ is the series resistance, $R_p$ is the parallel resistance, $X_p$ is the parallel reactance.
HELPFUL HINTS

Do not try to match more than a 4:1 impedance transformation. If you have to match over a larger impedance ratio, use two or more matching sections.

Before using a series L or series C, look at your design. If you need a d.c. blocking capacitor, use your matching C for this d.c. block. If you need to feed d.c. to a circuit component, use an L for this as part of the matching circuit.
HELPFUL HINTS (con’t)

Use complimentary L sections for broader bandwidth and higher impedance transformations; i.e. a two stage match.

Always match low to high impedances to simplify calculations. If you need to match a high impedance to a low impedance, do the calculations for low to high. Then remove the low impedance and connect the high impedance. The matching circuit is reciprocal and works both ways.
A series impedance has an equivalent parallel impedance. Both the series and parallel circuits have the same Q, and:

\[ Rp = Rs (Q^2 + 1) \]  (Remember this!!!)
EXAMPLE 1 – COMBINING FOUR ANTENNAS

Match 50 ohms, such as an antenna, to 200 ohms. Rs = 50 ohms, Rp = 200 ohms.

Calculate Q from Rp = Rs (Q^2 + 1).
Q^2 + 1 = (200/50) = 4,
Q^2 = 3, Q = 1.732
Since Q = Xs/Rs,
Xs = Q * Rs = (1.732 * 50) = 86.6 ohms.

For a series inductor, the circuit is 50 + j86.6 ohms.
Q also = Rp/Xp, and Rp = 200 ohms.
Therefore, Xp = Rp/Q = 200/1.732 = +j115.5 ohms.
COMBINING FOUR ANTENNAS (con’t)

\[ +j 86.6 \text{ ohms} + 50 \text{ ohms} = +j115.5 \text{ ohms} + 200 \text{ ohms}. \]

Now place \(-j115.5 \text{ ohms}\) in parallel with \(+j115.5 \text{ ohms}\) leaving only the 200 ohm real impedance.
COMBINING FOUR ANTENNAS (con’t)

Since $+j115.5\ \text{ohms}$ in parallel with $-j115.5\ \text{ohms} = \infty$, resulting in:

200 ohms
Final matching circuit

\[ +j86.6 \quad 50 \quad -j115.5 \quad = \quad 200 \ \Omega \]

Component calculation for \(+j86.6\) at 144 MHz:

\[ |86.6| = \frac{2\pi f L}{g2188 g2168} = 6.28 \times (144 \times 10^6) \times L \]

\[ L = 95.8 \text{ nH} \]

Component calculation for \(-j115.5\) at 144 MHz:

\[ |115.5| = \frac{1}{(2\pi f C)} = \frac{159 \times 10^{-3}}{144 \times 10^6} \times C \]

\[ C = 9.6 \times 10^{-12} = 9.6 \text{ pf} \]
COMBINING FOUR ANTENNAS (con’t)
COMBINING FOUR ANTENNAS SIMPLIFIED

Make all feedline the exact same length. Now match 12.5 Ω to 50 ohms using a series inductor and parallel capacitor.

\[ Rp = Rs (Q^2 + 1), \quad Q = 1.732, \quad Xs = 23.9 \text{ nH}, \quad C = 38.2 \text{ pf}. \]

\[ 12.5 \Omega \quad 23.9 \text{ nH} \quad 38.2 \text{ pf.} \quad = \quad 50 \Omega \]
COMBINING FOUR ANTENNAS SIMPLIFIED
WHEN IS YOUR INDUCTOR 23.9 nH?

\[ f_r = \frac{1}{2\pi\sqrt{LC}} \]

\[ f_r = \frac{159 \times 10^{-3}}{(\sqrt{23.9 \times 10^{-9}} \times 36 \times 10^{-12})} \]

\[ f_r = 154.6 \text{ MHz} \]

To account for 9 nH of internal lead inductance for the 36 pf capacitor:
\[ L = 29.4 \text{ nH} + 9 \text{ nH} = 38.4 \text{ nH}, \text{ and} \]
\[ f_r = 135.2 \text{ MHz} \]
WHEN IS YOUR INDUCTOR 23.9 nH, con’t?

\[ f_r = \frac{1}{2\pi\sqrt{LC}} \]

\[ f_r = \frac{159 \times 10^{-3}}{2\pi \sqrt{38.4 \times 10^{-9} \times 36 \times 10^{-12}}} \]

\[ f_r = 135.2 \text{ MHz} \]

\[ |X_L| = 2\pi f L = 6.28 \times 135.2 \times 10^6 \times 38.4 \times 10^{-6} \]

\[ |X_L| = +j 32.6 \]

\[ |X_C| = \frac{1}{2\pi f C} = \frac{159 \times 10^{-3}}{135.2 \times 10^6 \times 36 \times 10^{-12}} \]

\[ |X_C| = -j 32.6 \]

@ 135.2 MHz:  +j32.6 –j32.6 + 50 = 50 \Omega
EXAMPLE 2 – Multi step impedance transformation

Solid state PA device typically is \(\sim 4\ \Omega\). Since this is \(> 4:1\) \((50/4 = 12.5)\), use two impedance transformations and use complementary L sections: i.e. if first transformation is series L and parallel C, the second transformation should be series C and parallel L.

Also, the interstage impedance should be the “geometric mean” impedance for the same percentage step to the beginning and ending impedance values.
DETERMINE THE “GEOMETRIC MEAN” IMPEDANCE

The geometric mean impedance = $\sqrt{4 \times 50} = 14.14 \ \Omega$. The $1^{\text{st}}$ stage impedance ratio is $14.14/4 = 3.535$, and the $2^{\text{nd}}$ stage impedance ratio is also $50/14.14 = 3.535$; both less than 4:1!

$1^{\text{st}}$ stage transforms $4 \ \Omega$ to $14.14 \ \Omega$ with an inductor. 
$\text{Rp} = \text{Rs} \ (Q^2 + 1)$, $14.14 = 4(Q^2 + 1)$, $Q = 1.59$.
$Q = \frac{X_s}{\text{Rs}}$, $X_s = Q\times\text{Rs} = (1.59\times4) = +j6.36$.

$X_p = \frac{\text{Rp}}{Q} = 14.14/1.59 = +j8.9 \ \Omega$.
Therefore, use $-j8.9 \ \Omega$ to resonate $+j8.9 \ \Omega$.

\[ 4 \ \Omega + j6.36 \ \Omega - j8.9 \ \Omega = 14.14 \ \Omega \]
2\textsuperscript{nd} stage transformation to 50 ohms

2\textsuperscript{nd} step transforms 14.14 \(\Omega\) to 50 \(\Omega\) with a capacitor.

\(\text{Rp} = \text{Rs} \ (Q^2 + 1)\), \(50 = 14.14(Q^2 + 1)\), \(Q = 1.59\).

\(Q = \frac{\text{Xs}}{\text{Rs}}\), \(\text{Xs} = Q \times \text{Rs} = (1.59 \times 14.14) = -j22.5\)

\(\text{Xp} = \frac{\text{Rp}}{Q} = \frac{50}{1.59} = -j31.44 \ \Omega\)

Therefore, use \(+j31.44 \ \Omega\) to resonate \(-j31.44 \ \Omega\).
DETERMINE COMPONENT VALUES FOR 144 MHZ

+j6.36 \, \Omega = 7 \, \text{nH}
-j8.9 \, \Omega = 123.4 \, \text{pf}
-j22.5 \, \Omega = 49 \, \text{pf}
+j31.4 \, \Omega = 34.7 \, \text{nH}
EXAMPLE 3: USED TO DIVIDE POWER 2 WAYS FOR UNEQUAL LEVELS AND MAINTAIN 50 OHMS FOR EXCITER.

50 Ω, 100 watts  50 Ω, 40 watts

50 MHz Exciter  UNBALANCE DIVIDER  POWER AMPLIFIER

50 Ω  60 watts

Exciter has 100 watts output. PA requires 40 watts. Need to dissipate 60 watts into 50 Ω termination and still present 50 ohm load to the exciter.
UNBALANCED POWER DIVIDER, con’t

Determine impedances for desired power split.

\[ R = 50 \Omega = \frac{R_1 \cdot R_2}{R_1 + R_2} \]

\[ P = \frac{V^2}{R}, \quad P_1 = \frac{V^2}{R_1}, \quad P_2 = \frac{V^2}{R_2} \]

\[ 40 \, W \cdot R_1 = V^2 = 60 \, W \cdot R_2 \]

\[ R_1 = 1.5R_2 \]
\[ R = 50\Omega = \frac{R_1 R_2}{R_1 + R_2} \]

Substituting 1.5 \( R_2 \) for \( R_1 \)

\[ 50\Omega = \frac{1.5 R_2 R_2}{1.5 R_2 + R_2} \]

125\( R_2 = 1.5 \ R_2 \)

1.5\( R_2 = 125 \)

\[ R_2 = 83.3 \ \Omega \]

\[ R_1 = 125 \ \Omega \]

The 50 \( \Omega \) input to the PA should be matched to 125 \( \Omega \), and the 50 \( \Omega \) input to the termination should be matched to 83.3 \( \Omega \).
For the PA side to match 50 Ω to 125 Ω:
Rp = Rs (Q^2 + 1), 125 = 50(Q^2 + 1), Q = 1.22.
For a series capacitor, Xs = Q*Rs = (1.22*50) = -j 61, and
Xp = Rp/Q = 125/1.22 = -j102.5 Ω.

For the 50 Ω termination side to match 50 Ω to 83.3 Ω:
Rp = Rs (Q^2 + 1), 83.3 = 50(Q^2 + 1), Q = 0.82.
For a series inductor, Xs = Q*Rs = (0.82*50) = +j41, and
Xp = Rp/Q = 83.3/0.82 = +j101.6 Ω.

Since the parallel reactance toward the PA is –j 102.5
and the parallel reactance toward the termination is
+j101.6, i.e. approximately the same, these can be used
to compensate for each other with the parallel combination!
UNBALANCED POWER DIVIDER, con’t

\[ 50\Omega \quad \text{PA in} \]

\[ -j61 = 125 \quad -j102.5 \quad +j101.6 \quad 83.3 \quad = \quad +j41 \quad 50\Omega \quad \text{load} \]

\[ 125 \quad 83.3 \quad +j11,571 = 50\Omega \]

\[ 36.9\mu H \]
FINAL UNBALANCED POWER DIVIDER CKT.

@ 50 MHz:

-40 watts to PA

-j61 Ω

52 pf

-100 watts from transceiver

+j41 Ω

130 nH load

60 watts to 50 Ω

-40 watts to PA

-100 watts from transceiver

-20 watts to PA
EXAMPLE 4: GENERATION OF I AND Q SIGNALS

@ 144 MHz:

144 MHz Oscillator

50 Ω
+j50 in

Mini-Ckts GALI-55+

V₀ ↛ +45, I (0°)

-50 Ω
-j50 in

Mini-Ckts GALI-55+

V₀ ↛ -45, Q (90°)
I AND Q GENERATION, con’t

\[
\begin{align*}
50\Omega & \quad -j50 = 100 \\
 & \quad -j100 \quad +j100 \\
100 & \quad = \quad +j50 \\
\end{align*}
\]

\[
\begin{align*}
100 & \quad 100 = 50\Omega
\end{align*}
\]
@ 144 MHz:

144 MHz Oscillator

+j50 in
55 nH

-\j50 in
22 pf.

50 Ω

50 Ω

Mini-Ckts GALI-55+

V_o <+45, I (0°)

V_o <-45, Q (90°)
1296 MHz QUADRATURE COUPLER

@ 1296 MHz:

\[ 50 \Omega \]

\[ +j50 \Omega \]

\[ 6.1 \text{ nH} \]

\[ -j25 \Omega \]

\[ 25 \Omega \]

\[ \lambda/4 \]

\[ 50 \text{ OHM LINE} \]

\[ 50 \Omega \]

\[ P_0 \angle 0^\circ \]

\[ P_0 \angle -90^\circ \]

50 = 25(Q^2 + 1)

Q = 1 = Xs/Rs = Rp/Xp

Xs = -j25 \Omega

Xp = +j50 \Omega
1296 MHz QUADRATURE COUPLER
EXAMPLE 5: DESIGN INPUT MATCH FOR MINIMUM NF

Typical source impedance from min. NF contour on Smith Chart:

Min. NF source Impedance =
0.82 +j 0.18 (normalized)
= 41 + j9

∴ load impedance =
41 - j9, complex conjugate
Q = 0.22, Q^2+1 = 1.048
Rp = Rs(Q^2+1)
= 41*1.0484) = 43
Xp = Rp/Q = 43/0.22
= -j186
Use \( +j186 \) to resonate \( -j186 \). At 1296 MHz, \( +j186 = 0.023 \, \mu\text{H} \) or 23 nH, yielding:

\[
\begin{align*}
41 +j9 &= 43 -j186 \\
\text{Leaving} &
\end{align*}
\]

\[43 \, \Omega\]

This is sufficiently close to 50 \( \Omega \), or a \( \lambda/4 \) line \((\sqrt{50 \times 43} = 46.4 \Omega)\) can be added between the 43 \( \Omega \) and 50 \( \Omega \) input to achieve the 50 \( \Omega \) match. (This would be academic.)
SUMMARY

- **EXAMPLE 1:** Combining 4 antennas
- **EXAMPLE 2:** Two step impedance transformation for greater impedance ratios
- **EXAMPLE 3:** Unbalance power divider
- **EXAMPLE 4:** Generation of I and Q signals
- **EXAMPLE 5:** LNA input circuit for minimum noise figure
QUESTIONS?